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Transitions of the Néel vector in antiferromagnets

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Abstract. The quantum and classical transitions of the Néel vector in antiferromagnets are considered. Expressions for the tunnelling and thermal activation rates of the transitions are given for low temperatures, near the cross-over temperature and for high temperatures, so that they could be checked experimentally. Quantum and thermally activated nucleations in bulk solids and two-dimensional films of antiferromagnet are also presented.

1. Introduction

Interest in macroscopic quantum tunnelling (MQT) in magnetic systems has been increasing in the last few years [1–12]. The idea follows along the lines suggested by Caldeira and Leggett [13] for MQT in general [14] which has been considered extensively in systems of Josephson junctions [15–17], and superconducting quantum interference devices (SQUIDs) [18]. Quantum tunnelling of magnetization in ferromagnets was considered theoretically by Chudnovsky and Gunther [4], Garg and Kim [6] and Simanjuntak [9]. Experimental observations on quantum tunnelling of magnetization have also been reported by several groups [10–12]. More recent systems that have been considered are antiferromagnets where the formulation for quantum tunnelling of the Néel vector was performed independently by Krive and Zaslavskii [7] and Barbara and Chudnovsky [8]. In [8] the exponent of the rate of quantum coherence of the Néel vector was calculated with the pre-factor left incomplete. In [7] the exponent of the nucleation rate of the Néel vector was calculated using the thin-wall approximation. Since the thin-wall approximation is only good for the anisotropy ratio $\varepsilon \to 1$ (see below) which corresponds to very small rates, the results are in general difficult to demonstrate in experiments. The purpose of the present paper is to extend the calculation of the rates of the Néel vector transitions for a more general anisotropy ratio $\varepsilon > 1$ so that the results would be applicable to low, moderate and high rates to make them more accessible for experimental checks. We also present results for quantum and thermal nucleations.

We start with the Euclidean action for a system of antiferromagnet (neglecting dissipation) which is given by [7]

$$S[\theta(x,\tau)] = \int_{-\beta\hbar/2}^{\beta\hbar/2} \mathrm{d}\tau \int \mathrm{d}^3x \left\{ \frac{\chi_\perp}{2\gamma^2}\left[\left(\frac{\mathrm{d}\theta}{\mathrm{d}\tau}\right)^2 + \left(\frac{\mathrm{d}\phi}{\mathrm{d}\tau}\right)^2 \sin^2\theta\right] + \tfrac{1}{2}\alpha[(\nabla\theta)^2 + (\nabla\phi)^2\sin^2\theta] - U(\theta)\right\} \tag{1}$$

where $\gamma = ge/2mc$ (g is the g-factor). $\beta = 1/kT$ where k is the Boltzmann constant and T is the temperature. θ and ϕ are the angles in the spherical coordinate system with $\hat{l}\cdot\hat{z} = \cos\theta$; $\hat{l}$ is the Néel vector of unit length and $\hat{z}$ is a unit vector along the z direction. α is the exchange constant and $\chi_\perp$ is the *perpendicular* susceptibility of the antiferromagnet. The potential $U(\theta)$ is given by

$$U(\theta) = \tfrac{1}{2}K(\sin^2\theta - \varepsilon\sin^4\theta) \qquad \varepsilon = K_1/2K \tag{2}$$

where K and K_1 are anisotropy constants. We shall consider the case when $\varepsilon > 1$ where $\theta = 0$ is a metastable state and $\theta = \frac{1}{2}\pi$ is a stable state for the Néel vector.

The calculation of the rate Γ of transitions of the Néel vector from $\theta = 0$ to $\theta = \frac{1}{2}\pi$ can be performed as usual by the path integral method [19, 20]. That is, Γ takes the form

$$\Gamma = A\exp(-B/\hbar) \tag{3}$$

where $B = S[\theta_c(x,\tau)]$ and $\theta_c(x,\tau)$ is the classical solution of $\delta S[\theta_c(x,\tau)] = 0$. A is a pre-factor (see below). In the following we shall first consider the case of the homogeneous Néel vector and then continue to the nucleation problem.

2. Homogeneous Néel vector

In this section, we consider the case of the homogeneous Néel vector $\hat{l}$ and find the quantum tunnelling and thermal activation rates of transitions of $\hat{l}$ at constant ϕ. In this case, the imaginary-time action becomes

$$S[\theta(\tau)] = V\int_{-\beta\hbar/2}^{\beta\hbar/2} \mathrm{d}\tau \left[\frac{\chi_\perp}{2\gamma^2}\left(\frac{\mathrm{d}\theta}{\mathrm{d}\tau}\right)^2 + U(\theta)\right] \tag{4}$$

where V is the volume of a particle of the antiferromagnet. By using $\Omega \equiv (\gamma^2 K/\chi_\perp)^{1/2}$ and the dimensionless time $t \equiv \Omega\tau$ we can write equation (4) as

$$S[\theta(t)] = \frac{\chi_\perp}{\gamma^2}V\Omega\int_{-\beta\hbar\Omega/2}^{\beta\hbar\Omega/2} \mathrm{d}t \left[\frac{1}{2}\left(\frac{\mathrm{d}\theta}{\mathrm{d}t}\right)^2 + \tfrac{1}{2}(\sin^2\theta - \varepsilon\sin^4\theta)\right]. \tag{5}$$

With the action in equation (5), the path integral method gives the tunnelling rate as [13, 19–24]

$$\Gamma = \Omega\left(\frac{B_0}{2\pi\hbar}\right)^{1/2}\left[\int_{-\beta\hbar\Omega/2}^{\beta\hbar\Omega/2} \mathrm{d}t\left(\frac{\mathrm{d}\theta_c}{\mathrm{d}t}\right)^2\right]^{1/2}\left(\prod_n \lambda_n^0 \Big/ \prod_n{}' |\lambda_n|\right)^{1/2} \exp\left(\frac{-B}{\hbar}\right) \tag{6}$$

where B_0 is given by

$$B_0 = \frac{\chi_\perp}{\gamma^2}V\Omega. \tag{7}$$

The classical equation of motion $\delta S[\theta_c(t)] = 0$ is given by

$$-\frac{\mathrm{d}^2\theta_c(t)}{\mathrm{d}t^2} + \sin\theta_c\cos\theta_c - 2\varepsilon\sin^3\theta_c\cos\theta_c = 0 \tag{8}$$

with the boundary conditions $\theta_c(-\frac{1}{2}\beta\hbar\Omega) = \theta_c(\frac{1}{2}\beta\hbar\Omega)$ and $\mathrm{d}\theta_c(t)/\mathrm{d}t = 0$ at $t = \pm\frac{1}{2}\beta\hbar\Omega$. The eigenvalues λ_n^0 and λ_n in equation (6) satisfy the eigenvalue equations

$$\left(-\frac{\mathrm{d}^2}{\mathrm{d}t^2} + 1\right)\psi_n^0(t) = \lambda_n^0\psi_n^0(t) \tag{9}$$

and

$$\left(-\frac{\mathrm{d}^2}{\mathrm{d}t^2} + \cos(2\theta_c) + 2\varepsilon(1 - 4\cos^2\theta_c)\sin^2\theta_c\right)\psi_n(t) = \lambda_n\psi_n(t). \tag{10}$$

Finally, the prime on the product in equation (6) means that the zero eigenvalue $\lambda_1 = 0$ is omitted.

We find that the solution of equation (8) is given by

$$\theta_c(t) = \tan^{-1}[\tan\theta_1\,\mathrm{dn}(t\sqrt{\varepsilon - 1 + E}\tan\theta_1)] \tag{11}$$

where $\mathrm{dn}(u)$ is the delta amplitude elliptic function [25] and $\theta_1 = -\theta_4 > \theta_2 = -\theta_3$ are the roots of $\sin^2\theta - \varepsilon\sin^4\theta = E$. The integration constant E satisfies the periodicity condition of $\theta_c(t)$, i.e.

$$\frac{K(q)}{\sqrt{\varepsilon - 1 + E}\tan\theta_1} = \frac{\hbar\Omega}{2kT} \tag{12}$$

where $K(q)$ is the complete elliptic integral of the first kind [25] and $q = (\tan^2\theta_1 - \tan^2\theta_2)^{1/2}/\tan\theta_1$.

Having found the solution in equation (11) we now find the temperature dependence of the tunnelling rate. For tunnelling at *low* temperatures, we find that the solution leads to the exponent B in equation (6) as

$$B = V\Omega\frac{\chi_\perp}{\gamma^2}\left[1 - \frac{\varepsilon - 1}{2\sqrt{\varepsilon}}\ln\left(\frac{\sqrt{\varepsilon} + 1}{\sqrt{\varepsilon} - 1}\right) - \frac{8}{\varepsilon - 1}\exp\left(\frac{-\hbar\Omega}{kT}\right)\right] \tag{13}$$

which is valid for $\exp(-\hbar\Omega/kT) \ll \frac{1}{64}(\varepsilon - 1)$.

It is difficult to calculate the product of eigenvalues in equation (6) for general ε and T. However, for large ε and at $T = 0$ K we can use the small-θ approximation to equations (5) and (8) so that the solution becomes $\theta_c(t) = (\operatorname{sech} t)/\sqrt{\varepsilon}$ and the product of the eigenvalues can be found by the standard method of Langer [19] to give the product as $2\sqrt{3}$. Using this approximation, the tunnelling rate in equation (6) becomes

$$\Gamma = 2\sqrt{3}\Omega\left(\frac{B}{2\pi\hbar} - \frac{4\chi_\perp V\Omega^2\exp(-\hbar\Omega/kT)}{\pi(\varepsilon - 1)\gamma^2 kT}\right)^{1/2}\exp\left(\frac{-B}{\hbar}\right). \tag{14}$$

For the thermal activation rate at *high* temperatures $T \gg T_0$, where T_0 is the cross-over temperature between tunnelling and thermal activation (see below), the path integral method gives the thermal activation rate as [26]

$$\Gamma = \frac{kT_0}{\hbar}\left(\frac{\lambda_0^0}{|\lambda_0|}\right)^{1/2}\left(\prod_{n=1}\frac{\lambda_n^0}{\lambda_n}\right)\exp\left(\frac{-U_B}{kT}\right) \tag{15}$$

where U_B is the barrier height of the potential $VU(\theta)$ which is

$$U_B = \frac{KV}{8\varepsilon}. \tag{16}$$

λ_n^0 and λ_n, respectively are now given by [14, 23]

$$\lambda_n^0 = \left(\frac{2\pi nkT}{\hbar}\right)^2 + \Omega^2 \tag{17}$$

and

$$\lambda_n = \left(\frac{2\pi nkT}{\hbar}\right)^2 - \left(2 - \frac{1}{\varepsilon}\right)\Omega^2. \tag{18}$$

The cross-over temperature T_0 between tunnelling and thermal activation is given by [14, 27] $\lambda_1 = 0$, so that

$$T_0 = \frac{\hbar\Omega}{2\pi k}\left(2 - \frac{1}{\varepsilon}\right)^{1/2}. \tag{19}$$

Using equations (17) and (18), the product of eigenvalues in equation (15) can be found so that the thermal activation rate at $T \gg T_0$ is

$$\Gamma = \frac{kT_0}{\hbar}\frac{\sinh(\pi T_0/T\sqrt{2-1/\varepsilon})}{\sin(\pi T_0/T)}\exp\left(\frac{-U_B}{kT}\right). \tag{20}$$

We now consider the thermal activation rate at T close to but slightly *greater* than T_0. In this temperature regime the rate becomes [21, 23]

$$\Gamma = a\frac{\sqrt{\pi}kT_0}{2\hbar}\left(2 - \frac{1}{\varepsilon}\right)^{-1/2}[1 - \mathrm{erf}(x)]\exp x^2 \exp\left(\frac{-U_B}{kT}\right) f \tag{21}$$

where

$$x = \left(\frac{VK}{kT}\right)^{1/2}\frac{(1 - 1/2\varepsilon)[(T/T_0)^2 - 1]}{\{-22 + 15\varepsilon + 11/\varepsilon - 9(\varepsilon - 1)^2/\varepsilon[4(T/T_0)^2 - 1]\}^{1/2}} \tag{22}$$

and the factor a is given by

$$a = \left(\frac{VK}{kT}\right)^{1/2}\frac{(2 - 1/\varepsilon)[(T/T_0)^2 + 1]}{\{-22 + 15\varepsilon + 11/\varepsilon - 9(\varepsilon - 1)^2/\varepsilon[4(T/T_0)^2 - 1]\}^{1/2}}. \tag{23}$$

The factor f is the product of eigenvalues as in equation (15) but with n that now starts from $n = 2$ so that

$$f = \frac{1}{\pi}\frac{(T/T_0 + 1)(2 - 1/\varepsilon)^{1/2}}{(T/T_0)^2 + 1/(2 - 1/\varepsilon)}\sinh\left(\frac{\pi T_0}{\sqrt{2 - 1/\varepsilon}T}\right). \tag{24}$$

Another case that we consider is tunnelling at temperatures close to but *lower* than T_0. For the solution $\theta_c(t)$ in equation (11), we can show that the exponent $B \equiv S[\theta_c(t)]$ of the tunnelling rate in equation (6) is approximately given by

$$B = \frac{\hbar K V}{8\varepsilon k T}\left[1 - 2\left(2 - \frac{1}{\varepsilon}\right)^2 \delta\right] - \frac{\chi_\perp \Omega V}{2\gamma^2}\left[\frac{3\pi\varepsilon^2}{\sqrt{2}(2\varepsilon - 1)} - \frac{3\pi\varepsilon^{3/2}}{(2\varepsilon - 1)^{1/2}} + \pi\left(2 - \frac{1}{\varepsilon}\right)^{1/2}\right]$$
$$\times \ln\left(\frac{1 - (2/3\varepsilon)(2 - 1/\varepsilon)(T_0/T)}{1 - (2/3\varepsilon)(2 - 1/\varepsilon)}\right) - \frac{\chi_\perp \Omega V}{2\gamma^2}\left(\pi\sqrt{2 - \frac{1}{\varepsilon}} - \frac{3\pi\varepsilon^{3/2}}{2(2\varepsilon - 1)^{1/2}}\right)$$
$$\times \left(\frac{1}{1 - (2/3\varepsilon)(2 - 1/\varepsilon)(T_0/T)} - \frac{1}{1 - (2/3\varepsilon)(2 - 1/\varepsilon)}\right) \tag{25}$$

where

$$\delta = \frac{\frac{4}{3}(T_0/T - 1)}{1 - (2/3\varepsilon)(2 - 1/\varepsilon)(T_0/T)}. \tag{26}$$

The pre-factor, on the other hand, can be approximated by the pre-factor in equation (21) so that the tunnelling rate in the regime of temperature being considered becomes

$$\Gamma = a\frac{\sqrt{\pi} k T}{2\hbar}\left(2 - \frac{1}{\varepsilon}\right)^{-1/2}[1 - \mathrm{erf}(x)]\exp x^2 \exp\left(\frac{-B}{\hbar}\right) f. \tag{27}$$

This result agrees with equation (21) at $T = T_0$. Better still, we replace $B/\hbar$ by U_B/kT in equation (27).

3. The nucleation

Let us now move on to the nucleation problem in antiferromagnets. In this case, the action is given by equation (1) in general but, as before, we shall consider the case of nucleations at constant ϕ. The exponent of the nucleation rate was calculated by Krive and Zaslavskii [7] for $\varepsilon \to 1$ with the thin-wall approximation. Their result, therefore, is limited to very low rates where the unstable state appears almost as stable and thus is very difficult to observe experimentally. In the following, we shall extend the calculation for the more general case $\varepsilon > 1$ so that it is applicable to low, moderate and high rates.

As before, we also use $t \equiv \Omega\tau$ and define the dimensionless variable $y \equiv x\sqrt{K/\alpha}$ so that the action that we are considering takes the form

$$S[\theta(y, t)] = \frac{\chi_\perp}{\gamma^2}\Omega\left(\frac{\alpha}{K}\right)^{3/2}\int_{-\beta\hbar\Omega/2}^{\beta\hbar\Omega/2} \mathrm{d}t \int \mathrm{d}^3 y \left[\frac{1}{2}\left(\frac{\mathrm{d}\theta}{\mathrm{d}t}\right)^2 \right.$$
$$\left. + \frac{1}{2}\left[\left(\frac{\mathrm{d}\theta}{\mathrm{d}y_1}\right)^2 + \left(\frac{\mathrm{d}\theta}{\mathrm{d}y_2}\right)^2 + \left(\frac{\mathrm{d}\theta}{\mathrm{d}y_3}\right)^2\right] + \tfrac{1}{2}(\sin^2\theta - \varepsilon\sin^4\theta)\right]. \tag{28}$$

In the following, we shall consider only nucleations at $T = 0$ K for antiferromagnet systems with dimensions much greater than $\sqrt{\alpha/K}$. In this case, the classical solution $\theta_c(y, t)$ that minimizes the action is a spherical [28] bubble $\theta_c(u)$ with the variable

$$u = (t^2 + y_1^2 + y_2^2 + y_3^2)^{1/2} \tag{29a}$$

for a three-dimensional bulk solid, and the variable

$$u = (t^2 + y_1^2 + y_2^2)^{1/2} \tag{29b}$$

for a two-dimensional film. The action in equation (28) becomes

$$S[\theta(u)] = \frac{\chi_\perp \Omega \alpha}{\gamma^2 K} d_N \int_0^\infty \mathrm{d}u \, u^N \left[\frac{1}{2}\left(\frac{\mathrm{d}\theta}{\mathrm{d}u}\right)^2 + \tfrac{1}{2}[\sin^2\theta - \varepsilon \sin^4\theta]\right] \tag{30}$$

where $d_3 = 2\pi^2\sqrt{\alpha/K}$ for a three-dimensional ($N = 3$) bulk solid, and $d_2 = 4\pi D$ for a two-dimensional ($N = 2$) film of thickness D. The equation for the spherical bubble then satisfies

$$\frac{\mathrm{d}^2\theta_c}{\mathrm{d}u^2} + \frac{N}{u}\left(\frac{\mathrm{d}\theta_c}{\mathrm{d}u}\right) - \sin\theta_c\cos\theta_c - 2\varepsilon\sin^3\theta_c\cos\theta_c = 0 \tag{31}$$

with $\mathrm{d}\theta_c/\mathrm{d}u = 0$ at $u = 0, \infty$.

For the action in equation (30) the rate of quantum nucleation *per unit volume (or area)* in the case of three-dimensional (or two-dimensional) systems can be found by the usual path integral method as [19, 20]

$$\Gamma = \Omega\left(\frac{K}{\alpha}\right)^{N/2}\left(\frac{B}{2\pi\hbar}\right)^{(N+1)/2}\left(\prod_n \lambda_n^0 \Big/ \prod_n{}' |\lambda_n|\right)^{1/2} \exp\left(\frac{-B}{\hbar}\right). \tag{32}$$

Again, here $B = S[\theta_c(u)]$ while λ_n^0 and λ_n now satisfy their corresponding eigenvalue equation

$$\left(-\frac{\mathrm{d}^2}{\mathrm{d}t^2} - \nabla_y^2 + 1\right)\psi_n^0(y,t) = \lambda_n\psi_n^0(y,t) \tag{33a}$$

and

$$\left(-\frac{\mathrm{d}^2}{\mathrm{d}t^2} - \nabla_y^2 + \cos(2\theta_c) + 2\varepsilon(1 - 4\cos^2\theta_c)\sin^2\theta_c\right)\psi_n(y,t) = \lambda_n\psi_n(y,t). \tag{33b}$$

Here and throughout, we use the notation $\nabla_y^2 = \partial^2/\partial y_1^2 + \partial^2/\partial y_2^2 + \partial^2/\partial y_3^2$ for a three-dimensional bulk solid and $\nabla_y^2 = \partial^2/\partial y_1^2 + \partial^2/\partial y_2^2$ for a two-dimensional film.

We have solved equation (31) numerically for nucleations in two and three dimensions for various ε. As an example, we have plotted in figure 1 the solution for $\varepsilon = 2$ for a two-dimensional film. The solution for a three-dimensional bulk solid is identical and therefore not shown. Having found the solution, we then use it to calculate the exponent B of the corresponding nucleation rate. For the result, we have written the exponent B for a three-dimensional solid as

$$B = \frac{\pi^2\chi_\perp\Omega}{\gamma^2}\left(\frac{\alpha}{K}\right)^{3/2} b_3^{\mathrm{Q}}(\varepsilon) \tag{34}$$

where the reduced expoonent $b_3^{\mathrm{Q}}(\varepsilon)$ is shown in table 1 for various ε. For a two-dimensional film, on the other hand, the exponent is written as

$$B = \frac{2\pi\chi_\perp\Omega\alpha D}{\gamma^2 K} b_2^{\mathrm{Q}}(\varepsilon) \tag{35}$$

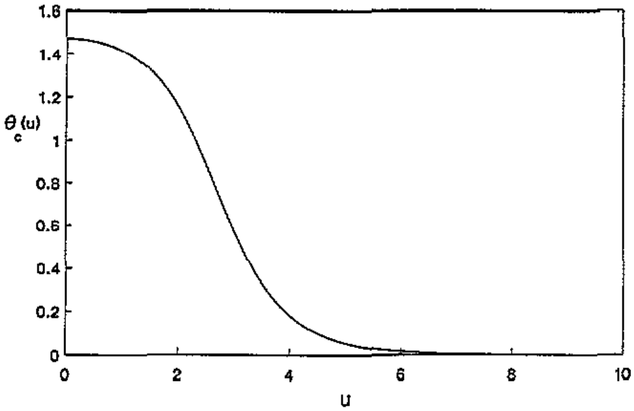

Figure 1. The solution $\theta_c(u)$ for quantum nucleations in a two-dimensional antiferromagnetic film with $\varepsilon = 2.0$.

Table 1. The reduced exponents for various ε. $b_3^Q(\varepsilon)$ is the reduced exponent for the rates of quantum nucleations in bulk solids. $b_2^Q(\varepsilon)$ is the reduced exponent for the rates of quantum nucleations in two-dimensional films. $b_3^T(\varepsilon)$ is the reduced exponent for the rates of thermal nucleations in bulk solids. $b_2^T(\varepsilon)$ is the reduced exponent for the rates of thermal nucleations in two-dimensional films.

ε	$b_3^Q(\varepsilon)$	$b_2^Q(\varepsilon) = b_3^T(\varepsilon)$	$b_2^T(\varepsilon)$
1.5	114.00	9.11	1.35
1.6	73.73	6.82	1.16
1.7	51.47	5.37	1.02
1.8	37.97	4.39	0.91
1.9	29.20	3.68	0.82
2.0	23.18	3.15	0.75
2.1	18.88	2.75	0.69
2.2	15.70	2.43	0.64
2.3	13.28	2.17	0.60
2.4	11.40	1.95	0.56
2.5	9.91	1.77	0.53
2.6	8.70	1.62	0.50
2.7	7.72	1.50	0.47
2.8	6.90	1.38	0.45
2.9	6.21	1.29	0.43
3.0	5.62	1.20	0.41
3.1	5.12	1.13	0.39
3.2	4.69	1.06	0.38
3.3	4.32	1.00	0.36
3.4	3.99	0.95	0.35
3.5	3.70	0.90	0.34
3.6	3.44	0.85	0.32
3.7	3.21	0.81	0.31
3.8	3.01	0.78	0.30
3.9	2.82	0.74	0.29
4.0	2.66	0.71	0.28

where the reduced exponent $b_2^Q(\varepsilon)$ is also given in table 1 for various ε. There still remains the product of eigenvalues in equation (33) that we have not solved.

We now consider thermal nucleations at T much greater than the cross-over temperature T_0 between quantum and thermal nucleations (see below). The action in equation (28) becomes

$$S[\theta(y)] = \frac{\chi_\perp}{\gamma^2}\left(\frac{\alpha}{K}\right)^{3/2}\beta\hbar\Omega^2\int \mathrm{d}^3y\,[\tfrac{1}{2}(\nabla_y\theta)^2 + \tfrac{1}{2}(\sin^2\theta - \varepsilon\sin^4\theta)] \tag{36}$$

and a bubble $\theta(y)$ has to reach a critical size before it can evolve. This critical size is determined by $\delta S[\theta_c(y)] = 0$. Since we are considering dimensions much greater than $\sqrt{\alpha/K}$, the critical bubble has the equation of motion as in equation (31) but with N that is now replaced by $N-1$, and u is like equation (29a) or (29b) but now without the variable t. For the temperature regime being considered (i.e. $T \gg T_0$), the path integral method leads to the thermal nucleation rate *per unit volume (or area)* for three (or two) dimensions as [29]

$$\Gamma = \frac{kT_0}{\hbar}\Omega\left(\frac{K}{\alpha}\right)^{N/2}\left(\frac{B}{2\pi\hbar}\right)^{N/2}\left(\prod_{\substack{n=1\\ \alpha}}\frac{\lambda^0_{n,\alpha}}{\lambda_{n,\alpha}}\right)\left(\prod_\alpha \lambda^0_{0,\alpha}\Big/\prod_\alpha{}'|\lambda_{0,\alpha}|\right)^{1/2}\exp\left(\frac{-B}{\hbar}\right). \tag{37}$$

Here we now have

$$\lambda^0_{n,\alpha} = \left(\frac{2\pi n}{\beta\hbar\Omega}\right)^2 + k^0_\alpha \tag{38a}$$

$$\lambda_{n,\alpha} = \left(\frac{2\pi n}{\beta\hbar\Omega}\right)^2 + k_\alpha \tag{38b}$$

where k^0_α and k_α satisfy the eigenvalue equations

$$(-\nabla^2_y + 1)Q^0_\alpha(y) = k^0_\alpha Q^0_\alpha(y) \tag{39a}$$

and

$$[-\nabla^2_y + \cos(2\theta_c) + 2\varepsilon(1-4\cos^2\theta_c)\sin^2\theta_c]Q_\alpha(y) = k_\alpha Q_\alpha(y). \tag{39b}$$

As before, the prime on the product in equation (37) means that the zero eigenvalues of equation (38b) are omitted.

We have also calculated the exponent B in equation (37) numerically for two and three dimensions. For this purpose, we have written B for a three-dimensional bulk solid as

$$B = \frac{2\pi\chi_\perp\hbar\Omega^2}{\gamma^2 kT}\left(\frac{\alpha}{K}\right)^{3/2} b^{\mathrm{T}}_3(\varepsilon) \tag{40}$$

where the reduced exponent $b^{\mathrm{T}}_3(\varepsilon)$ is shown in table 1 for various ε. Similarly, for a two-dimensional film,

$$B = \frac{\pi\chi_\perp\hbar\Omega^2\alpha D}{\gamma^2 kTK} b^{\mathrm{T}}_2(\varepsilon) \tag{41}$$

where the reduced exponent $b^{\mathrm{T}}_2(\varepsilon)$ is also shown in table 1 for various ε. Of course we need to find the cross-over temperature T_0 between quantum and thermal nucleations for the rate in equation (37). This temperature is determined by $\lambda_{1,0} = 0$ where k_0 is the ground-state (negative) eigenvalue of equation (39b). We have solved numerically the ground-state eigenvalue for various ε so that it leads to the cross-over temperature T_0 shown in table 2. There remains the products of eigenvalues in equation (37) that we have not solved.

Table 2. The reduced cross-over temperature for various ε. The second column is for a three-dimensional bulk solid and the third column is for a two-dimensional antiferromagnetic film.

	$2\pi kT_0/\hbar\omega_0$	
ε	3D bulk	2D film
1.5	0.09	0.19
1.6	0.12	0.26
1.7	0.15	0.34
1.8	0.19	0.43
1.9	0.23	0.52
2.0	0.27	0.61
2.1	0.31	0.71
2.2	0.36	0.80
2.3	0.40	0.90
2.4	0.45	0.99
2.5	0.50	1.08
2.6	0.55	1.18
2.7	0.60	1.26
2.8	0.65	1.35
2.9	0.70	1.44
3.0	0.76	1.52
3.1	0.81	1.60
3.2	0.87	1.68
3.3	0.92	1.75
3.4	0.98	1.83
3.5	1.03	1.90
3.6	1.09	1.97
3.7	1.15	2.03
3.8	1.21	2.10
3.9	1.26	2.16
4.0	1.32	2.22

4. Summary and discussion

We have presented the tunnelling and thermal activation rates of transitions of the Néel vector in antiferromagnets for various temperature regimes. The results for quantum and thermal nucleations in bulk solids and two-dimensional antiferromagnetic films are also given. As an illustration, for antiferromagnets with a homogeneous Néel vector with an anisotropy constant $K = 10^6$ erg cm^{-3}, a susceptibility $\chi_\perp = 10^{-4}$ and a particle radius $R = 30$ Å the rate of quantum transitions of the Néel vector would be 0.1 s^{-1} for $\varepsilon = 1.5$. The cross-over temperature T_0 between tunnelling and thermal activation in this case would be 2.4 K. For $\varepsilon = 2.5$ the rate would be 1.8×10^5 s^{-1} and the corresponding cross-over temperature would be 2.7 K. A larger particle size would require a higher ε for the rates to be appreciable. For example, with $R = 40$ Å and $\varepsilon = 3$ the rate would be 1.3×10^{-2} s^{-1} and $T_0 = 2.8$ K. For quantum and thermal nucleations, on the other hand, the cross-over temperature for a bulk solid with the given K and $\chi_\perp$ as above would be 0.2 K for $\varepsilon = 1.5$ and 1.1 K for $\varepsilon = 2.5$. The cross-over temperature in a two-dimensional antiferromagnetic film would be 0.4 K for $\varepsilon = 1.5$ and 2.3 K for $\varepsilon = 2.5$. It would be interesting now to see how our results can be manifested in experiments.

References

[1] For a review, see Stamp P C E, Chudnovsky E M and Barbara B 1992 *Int. J. Mod. Phys.* **6** 1355

[2] van Hemmen J L and Suto S 1986 *Europhys. Lett.* **1** 481
[3] Enz M and Schilling R 1986 *J. Phys. C: Solid State Phys.* **19** 1765, L711
[4] Chudnovsky E M and Gunther L 1988 *Phys. Rev. Lett.* **60** 661
[5] Caldeira A O and Furuya K 1988 *J. Phys. C: Solid State Phys* **21** 1227
Chudnovsky E and Gunther L 1988 *Phys. Rev.* B **37** 9455
[6] Garg A and Kim G-H 1989 *Phys. Rev. Lett.* **63** 2512
[7] Krive I V and Zaslavskii O B 1990 *J. Phys.: Condens. Matter* **2** 9457
[8] Barbara B and Chudnovsky E M 1990 *Phys. Lett.* **145A** 205
[9] Simanjuntak H 1993 *J. Low Temp. Phys.* **90** 405
[10] Paulsen C, Sampaio L C, Barbara B, Fruchard D, Marchand A, Tholence J L and Uehara M 1991 *Phys. Lett.* **161** 319
[11] Zhang X X, Balcells L I, Ruiz J M, Tholence J L, Barbara B and Tejada J 1992 *J. Phys.: Condens. Matter* **4** L163
[12] Awschalom D D, Smyth J F, Grinstein G, DiVincenzo D P and Loss D 1992 *Phys. Rev. Lett.* **68** 3092
[13] Caldeira A O and Leggett A J 1981 *Phys. Rev. Lett.* **46** 211; 1983 *Ann. Phys., NY* **149** 374
[14] For a review, see Hanggi P, Talkner P and Borkovec M 1991 *Rev. Mod. Phys.* **62** 251
[15] Voss R F and Webb R A 1981 *Phys. Rev. Lett.* **47** 697
Washburn S, Webb R A, Voss R F and Faris S M 1985 *Phys. Rev. Lett.* **54** 2712
Devoret M H, Martinis J M and Clarke J 1988 *Phys. Rev.* B **37** 5950
[16] Eckern U, Schön G and Ambegaokar V 1984 *Phys. Rev.* B **30** 6419
Larkin A I and Ovchinnikov Y N 1983 *Sov. Phys.–JETP* **58** 876
Simanjuntak H and Gunther L 1990 *J. Low Temp. Phys.* **78** 131
[17] Simanjuntak H and Gunther L 1991 *Phys. Rev.* B **42** 930
[18] den Boer W and de Bruyn Ouboter R 1980 *Physica* B **98** 185
Prance R J, Long A P, Clark T D, Widom A, Mutton J E, Sacco J, Potts M W, Megaloudis G and Goodall F 1980 *Nature* **289** 543
Bol D W, Scheffer J J F, Giele W T and de Bruyn Ouboter R 1985 *Physica* B **133** 196
Schwartz D B, Sen B, Archie C N and Lukens J E 1985 *Phys. Rev. Lett.* **55** 1547
[19] Langer J S 1967 *Ann. Phys., NY* **41** 108
[20] Coleman S 1977 *Phys. Rev.* D **15** 2929
Callen C G and Coleman S 1977 *Phys. Rev.* D **16** 1762
[21] Larkin A I and Ovchinnikov Y N 1984 *Sov. Phys.–JETP* **59** 420
[22] Hanggi P 1986 *J. Stat. Phys.* **42** 105
[23] Grabert H, Olschowski P and Weiss U 1987 *Phys. Rev.* B **36** 1931
[24] See, for example, equation (13) in section 2 of [21] and equation (6.7) in section VI.A of [23] or equation (9.39) in [14]
[25] Gradshteyn I S and Ryzhik I M 1980 *Tables of Integrals, Series and Products* (New York: Academic)
[26] See, for example, equation (4.9) in section IV.B of [23]
[27] Hänggi P, Grabert H, Ingold G-L and Weiss U 1985 *Phys. Rev. Lett.* **55** 761
[28] Coleman S, Glaser V and Martin A 1978 *Commun. Math. Phys.* **58** 211
[29] The derivation is analogous to the nucleation problem in a long Josephson junction in [17]